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SIMPLE FORMULAS AND GRAPHS FOR DESIGN OF VENTED LOUDSPEAKER SYSTEMS

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SIMPLE FORMULAS AND GRAPHS FOR DESIGN OF VENTED LOUDSPEAKER SYSTEMS

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This paper gives formulas for designing both fourth-order and sixth-order (equalized) systems for a given woofer. The formulas specify the frequency to which the box must be tuned, based on the woofer parameters. The designer has a choice of box volumes and cutoff frequencies (which are related). Graphs of the frequency response (versus box volume and woofer parameters) are also given. The graphs are scaled in units of f_s/Q_t and $V_{as}Q_t^2$, which makes them applicable to a very wide range of woofers and box sizes.

INTRODUCTION

The problem of finding the frequency response curve for a vented (bass reflex) loudspeaker system was first solved, for readers in this country at least, by the reprinting of Neville Thiele's classic paper "Loudspeakers in Vented Boxes" in the Journal of the Audio Engineering Society in 1971 (Ref. 1). His equation (12) gives the system frequency response function based on the physical parameters of the woofer and enclosure used.

Thiele simplified the problem by assuming that the enclosure Q (Q_b), that is, the Q associated with the resonance of the air mass in the
t with the compliance of the volume of air in the box, is infinite.
l gives the system response function taking into account the enclosure Q (Ref. 2, equation 13).

One might think that the "mother lode" of information provided by the system response function would have been pretty well mined out by now by the many investigators who have worked on this problem. Not so.

While measuring the properties of woofers that our company produces, I observed that while there was considerable unit-to-unit variation in the parameters of woofers of the same model, these variations had little effect on the acoustic performance of the systems in which they were used. This suggested to me that the performance of loudspeaker systems is actually determined by one or more relatively invariant properties of the woofers, while the commonly measured parameters defined above are unduly influenced by some varying factor that really has little effect on performance.

MATHEMATICAL ANALYSIS

The system frequency response function relates the relative sound pressure level output of a loudspeaker system, E(s), to the complex frequency variable, s. "Relative sound pressure level" means that the sound pressure level at high frequencies is assumed to approach unity, so that the function actually expresses the low-frequency output as compared to the higher-frequency output of the same driver-in-box. This corresponds to what is commonly called the "woofer section frequency response" of a multiple-driver speaker system. (This paper is solely concerned with the low-end response design problems.)

The response function is

$$E(s) = \frac{s^4}{\left[s^4 + \left(\frac{f_s}{Q_t} + \frac{f_b}{Q_b}\right)s^3 + \left(\frac{V_{as}f_s^2}{V_b} + f_s^2 + \frac{f_bf_s}{Q_bQ_t} + f_b^2\right)s^2 + \left(\frac{f_bf_s^2}{Q_b} + \frac{f_b^2f_s}{Q_t}\right)s + f_b^2f_s^2\right]} \qquad (1)$$

The frequency response is controlled by the six parameters in the function, three for the woofer and three for the box:

f_s natural resonant frequency of the woofer (measured on a flat baffle)

Q_t total Q of the woofer in the system at f_s, including all electrical and mechanical resistances, which I assume throughout this paper to equal Q_{ts}, the total Q of the woofer itself

V_{as} volume of air having same acoustic compliance as driver suspension

f_b frequency of enclosure resonance (resonance of air mass in vent with compliance of air volume in box)

Q_b total box Q at f_b due to all enclosure losses; assumed throughout this paper equal to 7

V_b net volume of air inside enclosure

The driver resonant frequency, f_s, is usually defined at the free-air (unenclosed) resonant frequency, but I prefer to use the resonant frequency as measured on a flat baffle because this method includes the effects of an air mass load more like the air load the driver experiences in an enclosure. Therefore frequency response calculations based on f_s defined and measured this way should be more accurate.

Under certain circumstances Q_t will differ from Q_{ts}. If damping material is packed close around the back of the woofer in the enclosure, the added resistance to air flow will change the woofer's effective Q. Also making the source impedance driving the woofer other than zero--for example by interposing resistance between the amplifier and woofer, or by designing as special negative-output-impedance amplifier--will change the value of Q (Ref. 5). However, ordinary high-damping-factor transistor amplifiers have near-zero output impedance, and so long as you have a low-resistance connection from the amp to the woofer the Q will be undisturbed; Q_t will equal Q_{ts}. (The marking "4-8 ohms" that you see on the back of amps means "attach a 4-8 ohm load here," not that the amplifier's output impedance is itself equal to 4-8 ohms.) I have assumed throughout this paper that Q_t and Q_{ts} are interchangeable. If you ever do run into a case where they differ, Q_t is the proper number to use in the formulas and graphs presented here.

The box Q, Q_b, is assumed here to be made up solely of leakage losses; losses of any other kind would change the form of eq. (1) slightly. I also assume throughout that Q_b is equal to 7. These two assumptions give results that are in reasonable conformity with practical speaker enclosure construction methods (Ref. 2).

There are other important driver parameters, such as the maximum power handling ability and the efficiency, that are not considered at all in this paper. Only factors that affect frequency response are considered. I would like to point out, however, that once a driver is selected, the enclosure design has no effect on system efficiency. The well-known dictum that "vented systems are more efficient than sealed systems" really means that a woofer optimized for use in a vented box of a certain size is more efficient than a woofer optimized for use in the same box sealed. This is because vented systems can use woofers of lower Q and still get good response shapes, and lower Q is caused by higher motor strength (larger magnet). Hence higher efficiency.

Keele observed that driver compliance V_{as} has relatively little effect on system frequency response (Ref. 3). I also know that of the woofer physical parameters--compliance, cone mass, magnet strength, and so on--compliance is the one that varies the most in production. An examination of driver resonant frequency and Q reveals that these two factors contain the compliance implicitly. The resonant frequency is sensitive to the compliance, of course, because it is determined by the cone mass and compliance. The Q is in turn influenced by the compliance because it is measured at the resonant frequency.

The undesired sensitivity to compliance may be "factored out" of the Q by choosing to describe the woofer's Q by

$$f_s/Q_t$$

rather than simply by Q_t. This factor may be analyzed by substituting the mechanical parameters for the acoustic parameters:

$$\frac{f_s}{Q_t} = \frac{2\pi\omega_s}{\omega_s M_{ms}/R_{ms}} = \frac{2\pi R_{ms}}{M_{ms}} \qquad (2)$$

ω_s is the resonant frequency in radians/second, R_{ms} is the total driver cone resistance in mechanical units, and M_{ms} is the cone mass in mechanical units. (For a complete description of the mechanical parameters and their effects, see Ref. 1 or 2.) You can see that the frequency, and therefore the dependence on compliance, cancels out.

Similarly we can use as a descriptive woofer parameter in place of V_{as} the factor:

$$V_{as}f_s^{\,2}$$

That this factor does not really have any dependence on compliance may again be demonstrated by expressing it in other parameters:

$$V_{as}f_s^{\,2} = V_{as}\left(\frac{1}{4\pi^2 V_{as}M_{as}}\right) = \frac{1}{4\pi^2 M_{as}} \qquad (3)$$

where M_{as} is the acoustic mass of the woofer. Thus the compliance cancels out and the factor $V_{as}f_s^{\,2}$ is really an expression of the acoustic mass of the woofer (or of the reciprocal of the mass, to be more exact).

Thus we may, if we wish, replace the three usual woofer parameters

$$f_s, \quad Q_t, \quad \text{and} \quad V_{as} \qquad (4)$$

with the "more fundamental" set

$$f_s/Q_t, \quad V_{as}f_s^{\,2}, \quad \text{and} \quad V_{as} \qquad (5)$$

The parameter V_{as} has no dependence on the other two so it can stay the same.

The "fundamentalness" of f_s/Q_t and $V_{as}f_s^2$ may be confirmed by re-examining the system response function, eq. (1). You can see that Q_t appears in three places, each time in the factor f_s/Q_t. The compliance V_{as} appears only once, and then in the factor $V_{as}f_s^2$.

This might suggest that loudspeaker parameters be specified in terms of the new set of factors (5) rather than the usual set (4). As a practical matter this is not necessary. Imagine that you have in hand a woofer manufactured with the correct cone mass and motor strength but the compliance is off by a certain amount. The true values of f_s, Q_t, and V_{as} for your woofer will all differ from the values specified by the manufacturer. However, the values of the factors f_s/Q_t and $V_{as}f_s^2$ as calculated from the specs will be correct for your woofer because it has the right cone mass and motor strength. Errors in f_s and Q_t caused by incorrect V_{as} will cancel when the parameters are combined in f_s/Q_t and $V_{as}f_s^2$. Your own calculation of these two factors will give you the same numbers as the manufacturer would have given you anyway.

To the woofer designer, however, for whom the motor strength, cone mass, and compliance are variables that he constrols independently, the new set of woofer parameters (5) may be very useful.

A BOLD CONJECTURE

After seeing how the new parameter set fit nicely into eq. (1), I turned to Thiele's well-known table of loudspeaker alignments (Table I, Ref. 1; with minor corrections in Ref. 3) to see if restating the column headings in terms of the new factors might lead to any simplifications. To my surprise I found on the right-hand side of the table a column of "approximately constant quantities" for the first nine Alignments. Thiele had observed that

$$\sqrt{\frac{V_{as}f_s^2}{V_bf_3^2}} = 1.41 \qquad (6)$$

and

$$\frac{Q_t}{f_s} f_b = .39 \qquad (7)$$

(My symbols here are slightly different than those in the original table.) f_3 is the -3 dB bass cutoff frequency.

Thus Thiele has uncovered two simple approximations relating f_3 and f_b, the frequency to which the box is tuned, to the three driver parameters and the box volume. Note that with Q_b assumed equal to 7, all six of the frequency-response-determining parameters in the system response function are accounted for. If we make the bold conjecture that this is all the information needed to design a vented speaker system, we can recast eq. (6) and (7) into two design formulas:

$$f_3 = \sqrt{\frac{V_{as} f_s^2}{1.41\ V_b}} = .84\sqrt{\frac{V_{as} f_s^2}{V_b}} \tag{8}$$

$$f_b = .39\frac{f_s}{Q_t} \tag{9}$$

The design procedure is very simple:

1) Pick a convenient size box, V_b.

2) Calculate the low-end cutoff (-3 dB) frequency it will give you, f_3, from eq. (8). If dissatisfied, go back to 1). This time pick a bigger box.

3) Tune the box resonant frequency, f_b, to the value specified by eq. (9).

The boldness of this conjecture is that it suggests that you can design a variety of vented systems, using different box volumes and getting different cutoff frequencies, for a given woofer. This is exactly what the loudspeaker system designer--who often has only a limited choice of drivers to pick from--would like to do. Most other design methods give only one allowable box volume, box frequency, and cutoff frequency for a given driver (unless you are willing to take extreme measures such as twiddling with Q_t by adjusting the source impedance).

FOURTH-ORDER VENTED SPEAKER SYSTEM RESPONSE GRAPHS

Formulas (8) and (9) can be tested by substituting the parameter values they prescribe back into the system response equation (1). We can use the response equation to see what sort of frequency response results from systems designed according to the formulas. The actual response calculations are very tedious and repetitive, but a programmable calculator handles them nicely.

The formula for f_b eliminates it as an independent variable, since it is now expressed in terms of other quantities. Assuming Q_b constant at 7 eliminates it as a variable. The number of variables may be further reduced by normalizing, that i., expressing some variables in terms of others.

A normalization scheme that works well is to express frequency in units of f_s/Q_t, that is, to use as the frequency variable

$$\frac{f}{f_s Q_t}$$

and to express volume in units of $V_{as}Q_t^2$, that is, to use as the volume parameter

$$\frac{V_b}{V_{as}Q_t^2}$$

The details of how and why this is done are explained in the Appendix.

The reduction in the number of variables simplifies the problem so that speaker systems designed for many combinations of f_s, Q_t, V_{as}, and V_b can have their frequency response characteristics expressed in a reasonable number of graphs. These graphs appear in Figures 1 through 7.

Each Figure is for one value of Q_t. Each Figure has several curves for different values of box volume V_b. The curves are labeled for volumes ranging from 2.0 $V_{as}Q_t^2$ to 16.0 $V_{as}Q_t^2$. The frequency scales are calibrated from .2 f_s/Q_t to 2.0 f_s/Q_t.

The unusual units will probably make it difficult for most people to conceptualize what the graphs represent. Imagine a "benchmark woofer" with

$$f_s/Q_t \quad = \quad 100 \text{ Hz} \qquad \text{and} \qquad V_{as}Q_t^2 \quad = \quad 1 \text{ liter or } 1 \text{ ft}^3$$

(A woofer with f_S = 15.9 Hz, Q_{TS} = .159, and V_{AS} = 39.6 liters or 39.6 ft^3 would fill the bill.)

For such a woofer you can read the frequency scale directly in hertz (ignoring the decimal points) from 20 to 200 hertz; and you can read the curves as though labeled simply in liters or ft^3.

To apply the curves to any other woofer:

1) Pick the Figure for Q_t value closest to that of the woofer you are considering.

2) Calculate the factor f_s/Q_t for the woofer and multiply the frequency scale numbers by that factor to get the frequency scale in herz.

3) Calculate the factor $V_{as}Q_t^2$ for the woofer and multiply the numbers that label the curves by that factor to get the curve labels in liters or ft^3.

Examination of these curves reveals some interesting facts. First of all, it is evident that for a given woofer you have a range of useable box volumes and bass cutoff (-3 dB) frequencies, rather than a single "correct" box volume. The cost of this flexibility is some ripple in the response curve. Even allowing only ±1 dB of ripple, however, the allowable box volumes span a considerable range. For example, a woofer with a Q of .32 can be used in boxes with volumes from 2.0 to 11.3 $V_{as}Q_t^2$ to give cutoff frequencies from .35 to .63 f_s/Q_t. This is a span of almost six to one in box volume, and two to one in cutoff frequency.

Also note that there is little difference in the curves from Figure to Figure; the curves for a woofer Q of .159 (Figure 1) are very similar to the curves for a woofer Q of .20 (Figure 2). This is because the normalization technique "factors out" the unwarranted sensitivity to compliance that injects large variations into families of response curves plotted using other systems of units.

This type of presentation also makes it possible for you to interpolate between the Figures for intermediate values of Q_t if you wish to do so.

The cutoff frequencies for various combinations of woofer Q and box volume are summarized in Table 1. The figures given in the body of the Table are limited to combinations that give ± 1 dB of ripple or less.

Remember that the curves in the Figures and the entries in Table 1 are true only for designs that conform to eq. (9). The fact that eq. (9) yields many designs with reasonable response shapes suggests that it is a worthwhile design tool.

The value of eq. (8) may be assessed by comparing the cutoff frequencies it predicts (which are given in Table 2) against the exact values for the cutoff frequencies calculated using the system response equation (which exact values are given in Table 1). Comparison shows that eq. (8)'s accuracy is rather poor.

Table 1

Cutoff frequencies (in units of f_s/Q_t) of fourth-order vented loudspeaker systems tuned according to eq. (9); for combinations of woofer Q and box volume which give ripple of ±1 dB or less.

	V_b (in units of $V_{as}Q_t^2$)						
Q_t	2.0	2.8	4.0	5.7	8.0	11.3	16.0
.159	f_3 = .61	.53	.46	.40	.37		
.20	.61	.53	.46	.40	.37	.35	
.25	.62	.54	.46	.40	.37	.35	
.32	.63	.55	.47	.40	.37	.35	
.40	.64	.56	.48	.41	.36	.35	
.50		.59	.51	.42	.36	.34	
.63			.57	.47	.35	.34	

Table 2

Estimates of cutoff frequency f_3 (in units of f_s/Q_t) of fourth-order vented loudspeaker systems tuned according to eq. (9). Estimates provided by eq. (8) (first row of the table) and eq. (10) (second row of the table). Comparison with Table 1 shows that eq. (10) is the better estimator.

	V_b (in units of $V_{as}Q_t^2$)						
	2.0	2.8	4.0	5.7	8.0	11.3	16.0
$f_3 = .84\sqrt{\dfrac{V_{as}f_s^2}{V_b}}$ =	.59	.50	.42	.35	.30	.25	.21
$f_3 = 1.0\sqrt{\dfrac{V_{as}f_s^2}{V_b}}$ =	.71	.59	.50	.42	.35	.30	.25

The main reason for the failure of eq. (8) is that it derives from figures based on the simplifying assumption that the box Q is infinite, while the curves in the Figures and the cutoff frequencies in Table 1 are calculated on the basis of box Q equal to 7. The finite box Q causes the vent radiation, and therefore the output in the region around cutoff, to be lower. This cause the frequency response curve to "sag" around the low end, and the -3 dB frequency to be higher. (Small gives alignment coefficients taking into account finite box Q in Ref. 2.)

A better estimator of cutoff frequency is

$$f_3 = 1.0\sqrt{\frac{V_{as}f_s^2}{V_b}} \qquad (10)$$

Cutoff frequencies predicted by this formula are also given in Table 2. Its accuracy is about ±15%.

FOURTH ORDER (VENTED) SPEAKER ENCLOSURE DESIGN EXAMPLE

The Speakerlab W804R is a 210-mm (8-inch) woofer generally used in a sealed enclosure of about 20 liters (0.7 ft^3). A high compliance unit with 8 ohms nominal impedance, its Thiele/Small parameters are:

f_s = 27 Hz
Q_t = .32
V_{as} = 99 liters (3.5 ft^3)

These give

f_s/Q_t = 84 Hz
$V_{as}Q_t^2$ = 10 liters (.36 ft^3)
f_b = $.39f_s/Q_t$ = 33Hz

The frequency response curves for Q_t equal to .32 appear in Figure 4. Figure 15 shows the same curves relabeled to correspond to the parameters of the W804R. Since $V_{as}Q_t^2$ is equal to 10 liters, the 1.4 curve has been relabeled 14 liters, the 2.0 curve has been relabeled 20 liters, and so on. Similarly, the frequency axis has been rescaled in hertz; 84 Hz takes the place of 1.0 f_s/Q_t, 168 Hz replaces 2.0 f_s/Q_t, etc.

The flattest-looking curve in Figure 15 is the 81 liter (2.8 ft^3) one, which gives response down to (-3 dB limit) 31 Hz. Increasing the box volume to 114 liters (4.0 ft^3) will reduce the cutoff frequency slightly to 30 Hz; decreasing the volume to 57 liters (2.0 ft^3) will raise the cutoff to 34 Hz. Even a box as small as 40 liters (1.4 ft^3) gives a cutoff of 40 Hz--not bad for a woofer this size.

Figure 15 also shows (dashed curve) the frequency response that results when the W804R is used in a design conforming to Thiele's Alignment 4. The Alignment 4 coefficients call for a box volume of 3.7 $V_{as}Q_t^2$, equal here to 53 liters (1.86 ft^3). Alignment 4 predicts (assuming infinite box Q) a cutoff frequency of 37 Hz; with a box Q of 7 (which Figure 15 assumes) the cutoff frequency is 41 Hz.

Butterworth alignments are designed to give "maximally flat" response, i.e., the response curve lying close below the 0-dB axis without crossing above it. Chebyshev alignments are designed to have equal ripple--equal humps above and below 0 dB.

Alignment 4 is a quasi-Butterworth alignment, and with infinite box Q would give response close to 0 dB above 50 Hz. But because my calculations and the curve in Figure 15 are based on box Q equal to 7, the curve sags about .3 dB around 70 Hz.

Figure 15 shows that confining your designs to systems that meet the Butterworth or Chebyshev criteria is unnecessarily restrictive. There are many other choices of box volumes that fail to meet those criteria because they have unequal humps and dips, but nevertheless give extended bass response within reasonable ripple limits.

The extended bass response is of course at the cost of box volume. It is up to the system designer to decide whether a given reduction in cutoff frequency is worth the extra volume required. Thiele's alignments, particularly if corrected for finite box Q as in Ref. 2, are optimum in this sense: smaller enclosures than called for by the alignment suffer rapid decay of bass response, while larger enclosures give small improvements in cutoff frequency at great cost in volume.

Frequency response curves for the W804R in sealed enclosures are shown in Figure 17 for comparison. The flattest curve is provided by a volume of 20 liters (.7 ft^3). Such a system gives response down only to 60 Hz. Venting the same enclosure (referring again to Figure 15) would extend the cutoff to 53 Hz.

Note that using a larger volume for the sealed system does not improve the frequency response much. While the curve for 40 liters stretches far to the left, response at 40 Hz is 6 dB down. The output level around 40 Hz could be brought up and the response made flatter by increasing Q_t (and increasing f_s and decreasing V_{as} so as to keep f_s/Q_t and $V_{as}Q_t^2$ constant). This could be done by weakening the magnet--at a savings in magnet material and a cost in lost efficiency. This is an example of how vented systems can be more efficient in comparison to sealed systems. There is a tradeoff between bass response and efficiency, and vented systems drive a harder bargain with the laws of acoustics.

SIXTH-ORDER (EQUALIZED) VENTED LOUDSPEAKER SYSTEM DESIGN

Fourth-order vented systems using high-compliance woofers are quite susceptible to excessive cone excursion caused by subsonic signals. The accepted wisdom that vented systems exhibit less cone excursion than sealed systems is true only with respect to in-band signals.

At very low frequencies, cone motion in both the sealed and vented cases is limited only by the total stiffness (reciprocal of the compliance). For the sealed system, the total stiffness is the sum of the cone suspension stiffness plus the enclosure air volume stiffness. The vented system, however, allows air flow in and out of the enclosure (the impedance of the vent air mass approaches zero with decreasing frequency) so that the only cone-restaining stiffness is that of the cone's own suspension.

My experience has been that nearly all record-playing equipment available today emits considerable subsonic signal energy at the output. This means that fourth-order systems using a high compliance woofer, such as the one in the previous design example, must either be operated at a reduced power level (compared to the power rating based on ability to handle in-band signals) or be operated with an auxiliary highpass filter inserted somewhere in the signal path to reduce the amplitude of subsonic signals.

Most home audio equipment, and of course all professional equipment, does allow for the insertion of extra processing in the signal path. In home systems a filter or equaliizer can go between the preamplifier and amplifier, or in the tape monitor circuit all modern preamplifiers and stereo receivers provide.

The addition of a filter or equalizer (Ref. 6) also makes possible the design of speaker systems with system response functions of higher order. Keele has pointed out that a fourth-order design can be converted to a sixth-order design by reducing the box frequency f_b by half an octave and using a second-order highpass filter with a Q of 2 (Ref. 4). The theoretical cutoff frequency of the filter, f_{aux}, is set the same as the new box frequency. The filter's frequency response exhibits a peak of 6 dB, which approximately offsets the sag in response of the driver/enclosure combination caused by the changed box tuning. Hopefully all this will result in an extension of the bass response by another half octave, as well as eliminate the undesired subsonic energy.

Encouraged by our previous success, we courageously write a new design formula:

$$f_b = f_{aux} = .276\frac{f_s}{Q_t} \tag{11}$$

I recalculated all the system response curves, this time with the system response function of eq. (1) multiplied by the second-order response function of the filter, and with f_b and f_{aux} as called for by eq. (11). The results appear in Figures 8 through 14. Comparison of these curves to those for the fourth-order case--and direct comparison is possible with the units defined as they are in the normalization used here--shows that detuning the box frequency one-half octave and using the equalizer filter does indeed extend the bass response considerably. However, the ripple is greater. The usable box volumes (giving ripple of ±1 dB or less) span a narrower range. There are no box volumes, in fact, that give ripple less than 1 dB for woofers with Q of .5 or over. Table 3 lists the cutoff frequencies for the useable combinations.

A useful approximation for estimating the cutoff frequency is

$$f_3 = .71 \frac{V_{as} f_s^{\ 2}}{V_b} \qquad (12)$$

Cutoff frequencies predicted by this approximation are listed at the bottom of Table 3 for comparison. The accuracy of the approximation is about ±20%.

SIXTH-ORDER (EQUALIZED) VENTED SPEAKER SYSTEM DESIGN EXAMPLE

The W804R woofer of the previous example may be used in a sixth-order system if per eq. (11) we set

$$f_b = f_{aux} = 23 \text{ Hz}$$

Figure 16 shows the sixth-order frequency response curves for a woofer with Q of .32, again rescaled to correspond to the parameters of the W804R.

The flattest curve this time is the one for 57 liters (2.0 ft^3), which gives a cutoff frequency of 23 Hz. Compare this to the flattest curve (81 liters) for the fourth-order design (Figure 15). Going to a sixth-order system extends bass response 26% lower in a 30% lower box, as well as improving the power handling ability and reducing distortion by reducing subsonic cone motion.

A box as small as 29 liters (1.0 ft^3) may be used for a 41 Hz cutoff, though at the cost of a +1.2 dB hump around 110 Hz.

Figure 16 also shows the frequency response of this woofer in a system designed to Thiele's Alignment 16, which the woofer fits well--the woofer's Q of .32 is almost exactly equal to the Q called for by the Alignment, .317. (In acoustics, ±1% is exact.) Note that Alignment 16 gives a nicely shaped response curve, though it has some sag because of the finite box Q used in calculating the curve. This also makes the cutoff frequency 25 Hz rather than 23 Hz as predicted by the Alignment.

Table 3

Cutoff frequency f_3 (in units of f_s/Q_t) versus box volume V_b (in units of $V_{as}Q_t^2$) of sixth-order (equalized) vented loudspeaker systems tuned according to eq. (11); for combinations of woofer Q and box volume which give ripple of ±1 dB or less. The bottom line of this table gives estimates of f_3 as provided by eq. (12).

			V_b (in units of $V_{as}Q_t^2$)				
Q_t	2.0	2.8	4.0	5.7	8.0	11.3	16.0
.159	f_3 = .55	.44	.32	.28*			
.20	.56	.45	.32	.28			
.25	.57	.47	.32	.28			
.32	.58	.48	.34	.28	.26		
.40	.60	.51	.39	.28	.25		

*This combination gives ripple greater than 1 dB--it gives a 1.1 dB dip.

Est. f_3 = $.71\sqrt{\dfrac{V_{as}f_s^2}{V_b}}$	= .50	.42	.36	.30	.25		

CONCLUSION

The box resonant frequency design rule for fourth-order vented speaker systems (eq. 9) gives designs with reasonably flat, extended bass response and can be applied over a wide range of driver parameters and enclosure volumes.

The design rule for sixth-order (equalized) systems (eq. 11) also works well, though over a narrower range of driver/enclosure combinations. A sixth-order design can provide bass cutoff as much as one-half octave lower than a fourth-order design using the same driver in the same volume enclosure.

Useful estimates of the cutoff frequency f_3 for fourth-order and sixth-order designs are provided by equations (10) and (12), respectively. More exact values for the cutoff frequency appear in Tables 1 and 3.

Actual frequency response curves for systems conforming to the design rules for f_b, and conforming to the assumption that box Q, Q_b, is equal to 7, appear in Figures 1 through 14. You can apply these response curves to systems with any arbitrary values of Q_t, f_s, V_{as} and V_b within reasonable limits. You can get response curves for values of Q_t and V_b between the values on which the curves are based by interpolation; you can interpolate between Figures as well as between curves in the same Figure.

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This paper follows on the work of many other people. I would particularly like to express my debt to D.B. Keele, Jr., who gave me his unpublished notes on computer plotting of frequency response using the system response equation at the 1976 International Loudspeaker Symposium in Colorado Springs. I am also grateful to J.R.Ashley and R.H.Small, speakers at and organizers of the Symposium. All the authors in the References appeared there and I received many useful ideas there.

APPENDIX--NORMALIZATION

Normalizing is a method of redefining the units (hertz and liters) appearing in a set of equations so as to simplify the equations. The normalization scheme used in this paper is chosen to make (in the normalized units)

$$\frac{f_s}{Q_t} = 1 \tag{13}$$

$$V_{as} f_s^2 = 1 \tag{14}$$

This is achieved by defining

$$f' = \frac{f}{f_s/Q_t} \tag{15}$$

$$V' = \frac{V}{V_{as} Q_t^2} \tag{16}$$

$$Q' = Q \tag{17}$$

where primed symbols denote normalized values and unprimed symbols denote the actual values in hertz and liters. Note that the normalized quantitites are dimensionless, i.e., pure ratios. *Q* stays the same because it was a dimensionless quantity to start with.

In normalized units the six parameters that determine frequency response in a fourth-order system become:

$$f_s' = \frac{f_s}{f_s/Q_t} = Q_t = Q_t' \tag{18}$$

$$V_{as}' = \frac{V_{as}}{V_{as} Q_t^2} = \frac{1}{Q_t^2} = \frac{1}{Q_t'^2} \tag{19}$$

$$Q_t' = Q_t \tag{20}$$

$$f_b' = .39\frac{f_s}{Q_t'} = .39(1) = .39 \tag{21}$$

$$V_b' = \frac{V_b}{V_{as} Q_t^2} \tag{22}$$

$$Q_b' = Q_b = 7 \qquad (23)$$

For sixth-order systems, we have

$$f_b' = f_{aux}' = .276\frac{f_s}{Q_t'} = .276 \qquad (24)$$

Thus normalizing helps reduce our six original independent parameters to two--Q_t' and V_b'.

To translate from the normalized units in the Figures back to real units of hertz and liters, we merely "denormalize," that is, multiply by f_s/Q_t and $V_{as}Q_t{}^2$ where previously we divided:

$$f = f'(f_s/Q_t) \qquad (25)$$

$$V_b = V_b'(V_{as}Q_t{}^2) \qquad (26)$$

To remind you to do this multiplication, I wrote "2.0 $V_{as}Q_t{}^2$" on one curve of each Figure. "2.0" is the value of V_b' on that curve.

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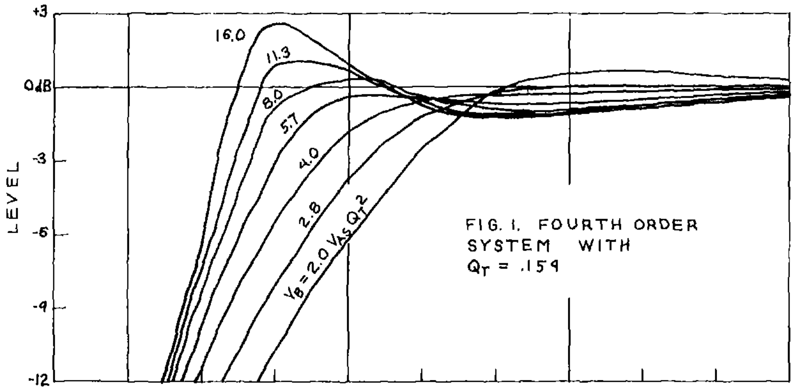

FIG. 1. FOURTH ORDER SYSTEM WITH Q_T = .159

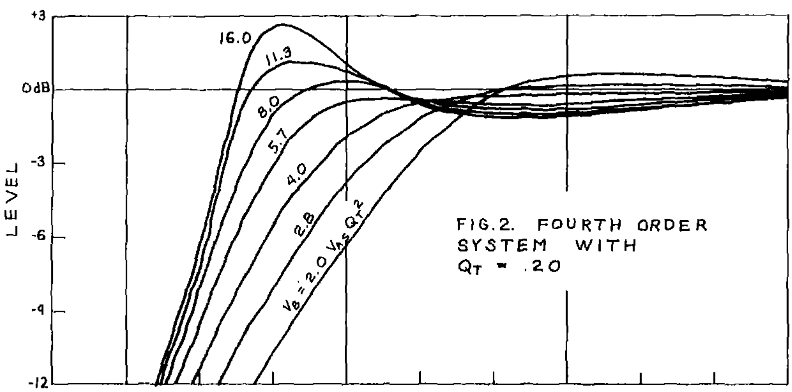

FIG. 2. FOURTH ORDER SYSTEM WITH Q_T = .20

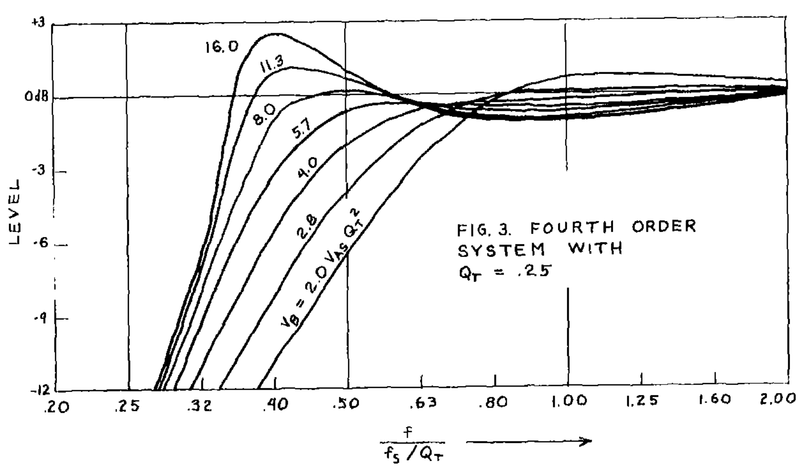

FIG. 3. FOURTH ORDER SYSTEM WITH Q_T = .25

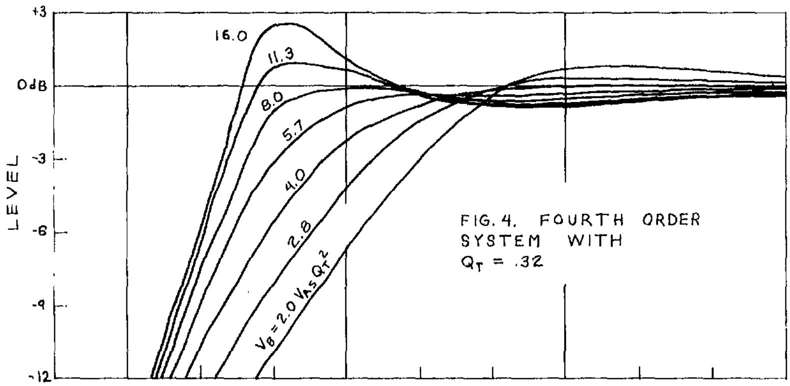

FIG. 4. FOURTH ORDER SYSTEM WITH $Q_T = .32$

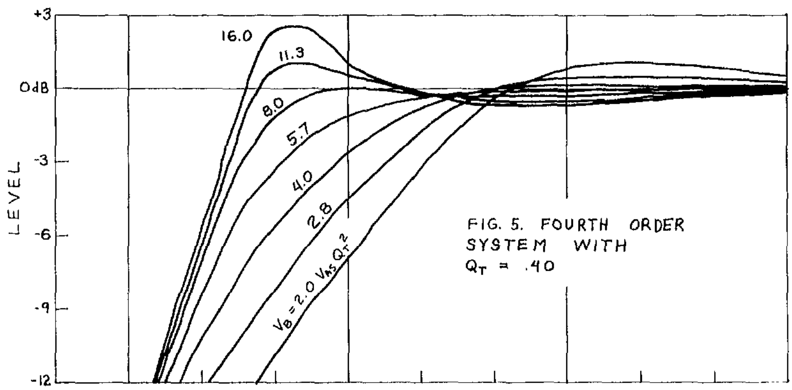

FIG. 5. FOURTH ORDER SYSTEM WITH $Q_T = .40$

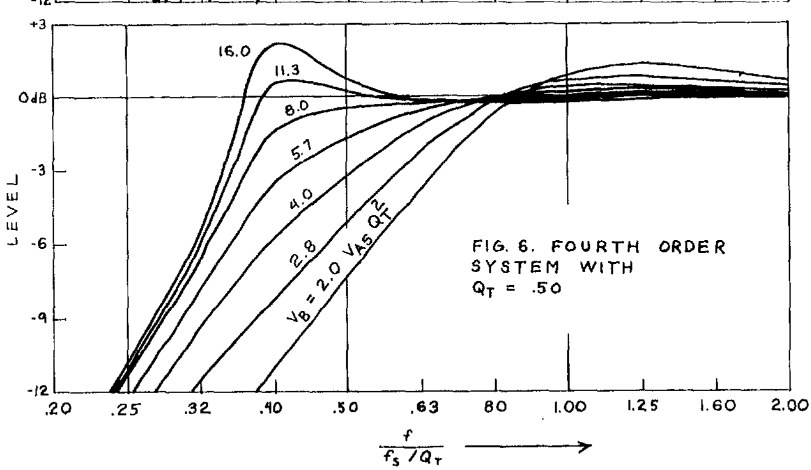

FIG. 6. FOURTH ORDER SYSTEM WITH $Q_T = .50$

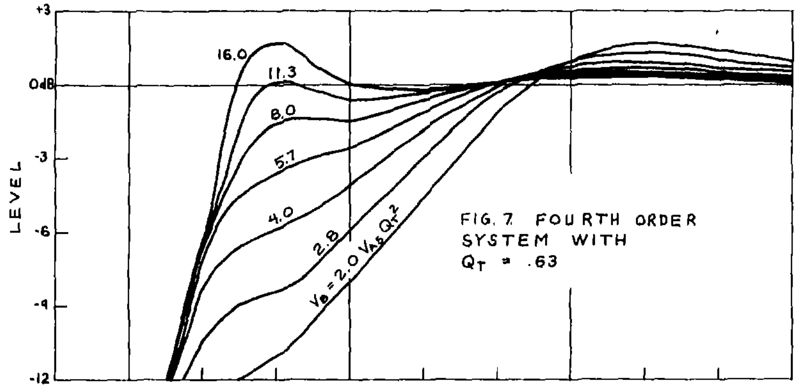

FIG. 7 FOURTH ORDER SYSTEM WITH Q_T = .63

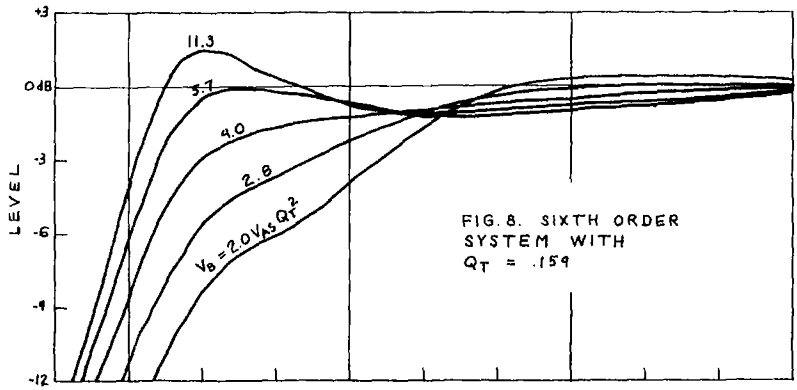

FIG. 8. SIXTH ORDER SYSTEM WITH Q_T = .159

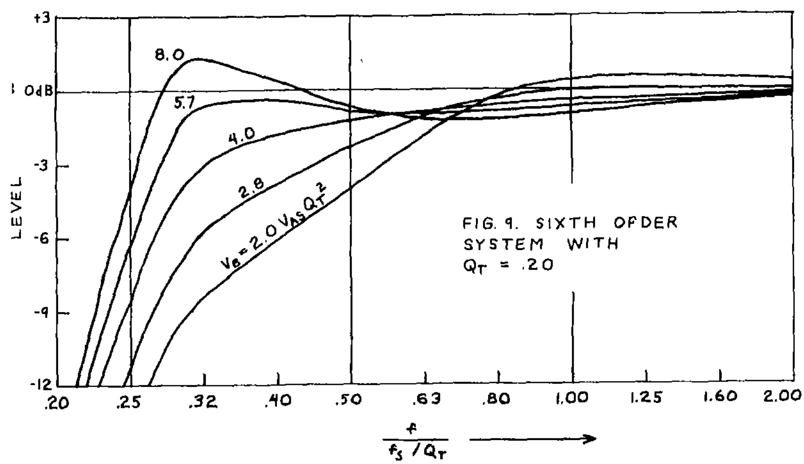

FIG. 9. SIXTH OFDER SYSTEM WITH Q_T = .20

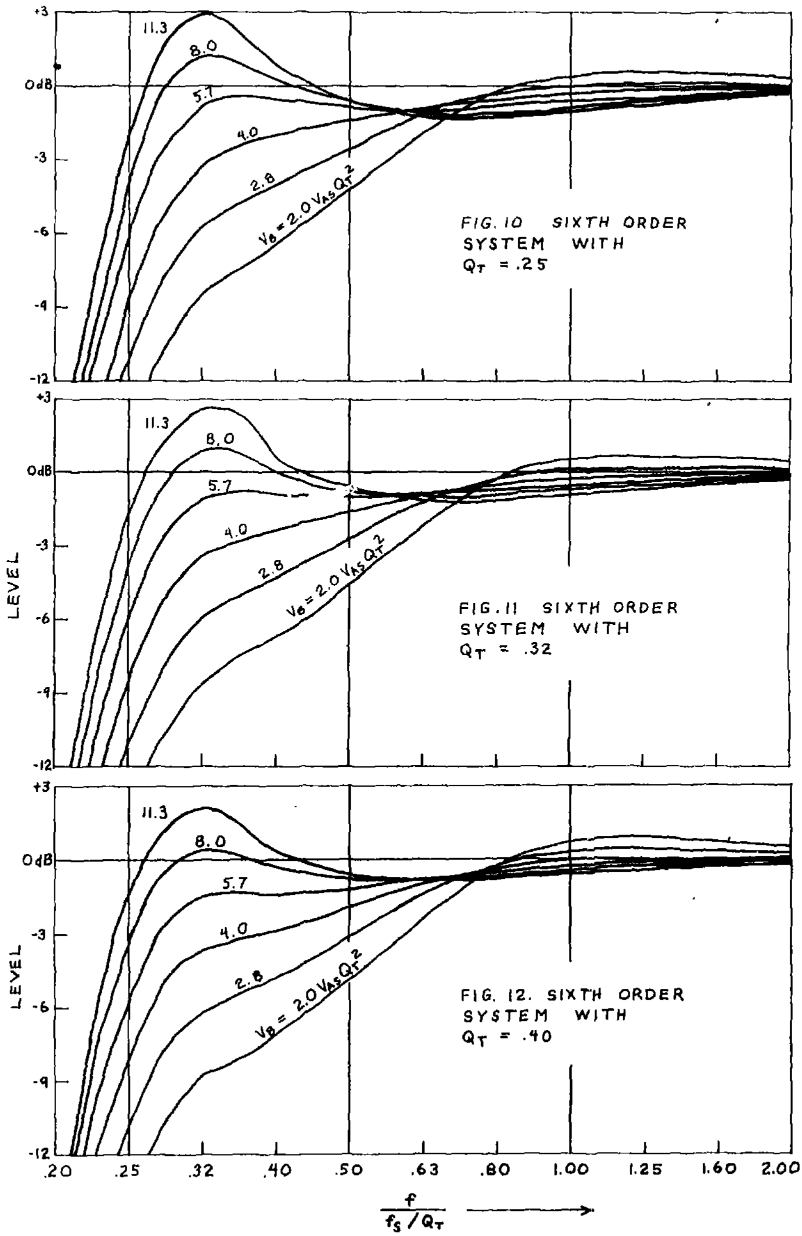

FIG. 10 SIXTH ORDER SYSTEM WITH $Q_T = .25$

FIG. 11 SIXTH ORDER SYSTEM WITH $Q_T = .32$

FIG. 12. SIXTH ORDER SYSTEM WITH $Q_T = .40$

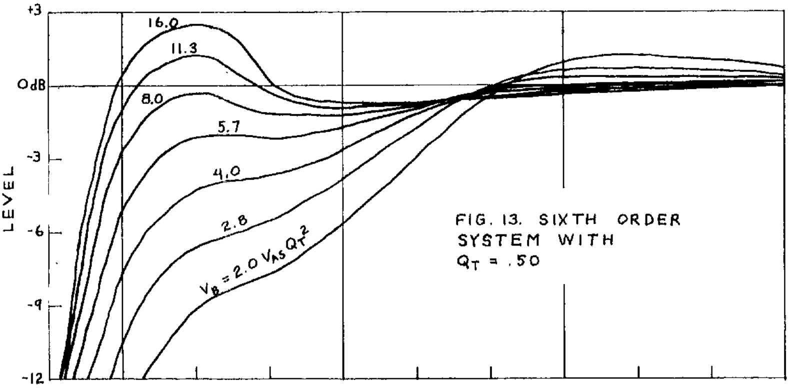

FIG. 13. SIXTH ORDER SYSTEM WITH $Q_T = .50$

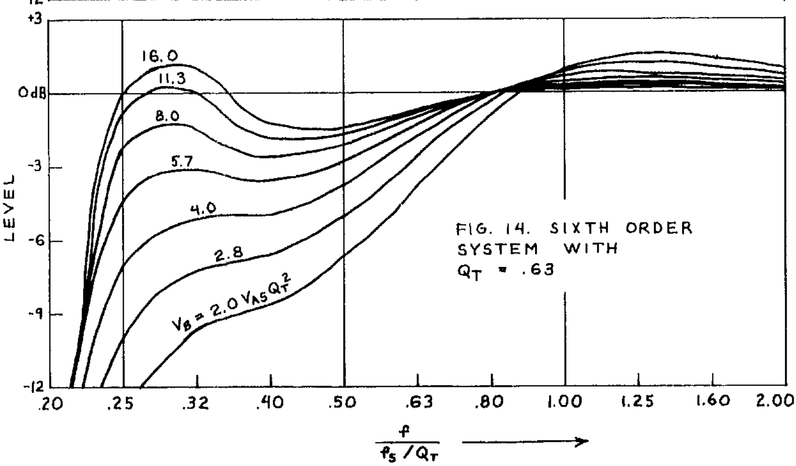

FIG. 14. SIXTH ORDER SYSTEM WITH $Q_T = .63$

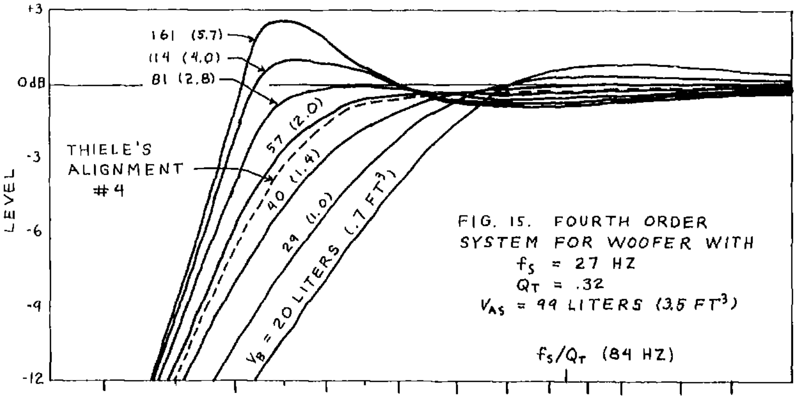

FIG. 15. FOURTH ORDER SYSTEM FOR WOOFER WITH
f_S = 27 HZ
Q_T = .32
V_{AS} = 99 LITERS (3.5 FT^3)

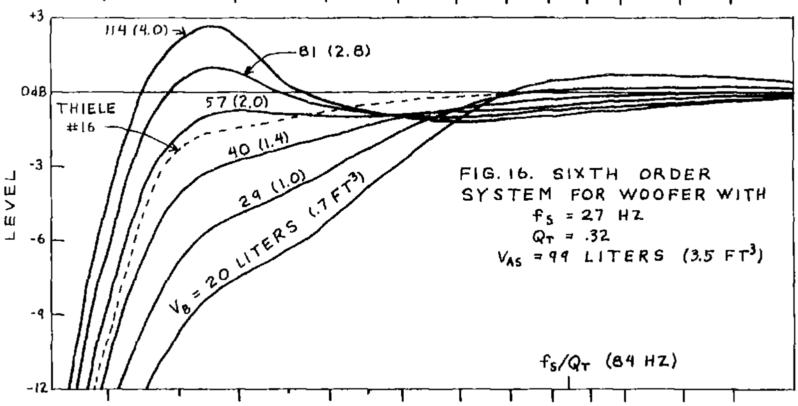

FIG. 16. SIXTH ORDER SYSTEM FOR WOOFER WITH
f_S = 27 HZ
Q_T = .32
V_{AS} = 99 LITERS (3.5 FT^3)

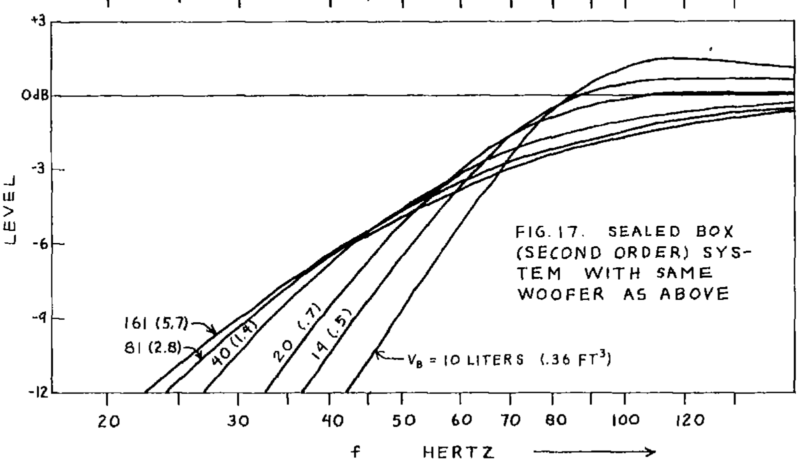

FIG. 17. SEALED BOX (SECOND ORDER) SYSTEM WITH SAME WOOFER AS ABOVE